

Readings: Read last sets of lecture notes and skim chapters 3 and 4 of Kitaev notes. If you feel ambitious, either read Read and Green paper from course webpage or read chapter on conformal invariance in Cardy, “Scaling and renormalization in statistical physics”, Cambridge UP.

Problems: There are only 4 problems this time so that you can spend some time studying the last of the course material. (Do any 2 for 2 units)

1. Justify in your own words the three gauge fixings that were essential in rewriting the pure Chern-Simons theory on a torus as a particle in a magnetic field, as follows. (1) Why does  $a_0 = 0$  imply a constraint? What is this constraint? (2) What additional gauge transformation should be made to impose  $\partial_i a_i = 0$ ? (3) What was the point of writing the last gauge transformation, which brings the particle motion from the plane to the torus, in the unwieldy form

$$a_\mu \rightarrow a_\mu - iu^{-1}\partial_\mu u \tag{1}$$

rather than simply using the gradient of a function  $\theta$ ?

2. Show by explicit construction that the one-dimensional Dirac equation has a localized zero-energy mode if the mass changes sign. Hint: think about  $\exp(-\int |m| dx)$ .

3. Show that the toric code (Kitaev) model on the plane is “ergodic”: any state with zero vortices (no bad magnetic plaquettes) can be reached from any other state satisfying this constraint by repeatedly acting with one of the terms in the Hamiltonian.

4. Show that the following Kitaev-like model with  $s = 1/2$  spins on the *sites* of a square lattice is self-dual with  $K \rightarrow h$ :

$$H = -K \sum_{\square} \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z - h \sum_i \sigma_i^x. \tag{2}$$

Here the first term refers to the spins at the four corners of a unit square, and the second term is just an ordinary magnetic field. Hint: in class we did the self-duality of the 1D quantum Ising model

$$H = - \sum_i (K \sigma_i^z \sigma_{i+1}^z + h \sigma_i^x) \tag{3}$$

using a “string” of operators to the left of a given site. Think about generalizing the string from 1D to 2D.